HABERDASHER'S PROBLEM BY DUDENEY

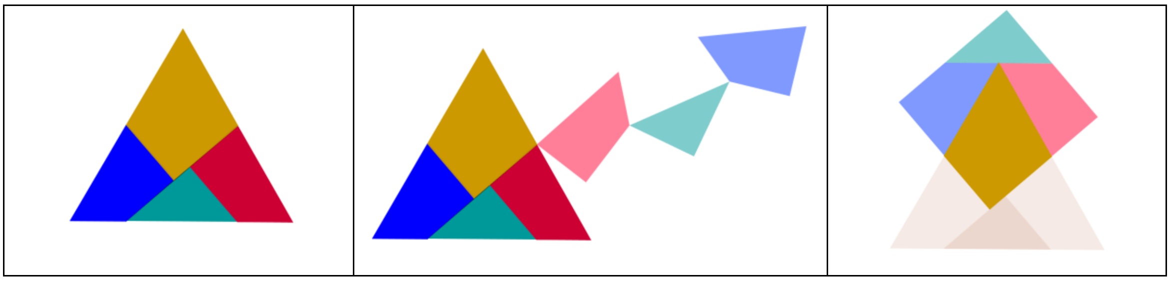
STUDENT'S WORKSHEETS AND SOLUTIONS

As a student, you have to carry out a research study independently. You will receive support and explanations from your teacher. In this case, your assignment deals with an old problem from Dudeney's puzzles. You will discover that there is little or no useful information to be found about this issue; it is an example of a valuable research project.

Here is a brief description of Dudeney and the puzzle he poses:
Henry Ernest Dudeney (1857-1930) was an English mathematician and inventor of some particularly famous puzzles. He was sixteen years younger than his fellow puzzle designer, the American Sam Loyd. In the late 19th century, they worked on a series of puzzle articles for *Tit-Bits* magazine and later exchanged a lot of puzzle problems for their magazine and newspaper columns. One of the most famous mathematical puzzles, the haberdasher's problem created by Dudeney in 1905, was presented to the Royal Society in 1908 and published in *Canterbury Puzzles*.

Cut an equilateral triangle into four pieces that can be rearranged to make a square with the same area.

A remarkable feature of the solution is that each of the pieces can be hinged at one vertex, forming a chain that can be folded into either the square or the original triangle (Such a puzzle is also called a hinge puzzle).



PART 1: AN INTRODUCTORY OVERVIEW

Henry Ernest Dudeney (1857-1930) was an English mathematician and inventor of some particularly famous puzzles. Dudeney was sixteen years younger than his fellow puzzle designer, the American Sam Loyd. In the late 19th century they worked on a series of puzzle articles for the Tit-Bits magazine and later they exchanged a lot of puzzle problems for their magazine and newspaper columns. One of the most famous mathematical puzzles, the Haberdasher’s Problem created by Dudeney in 1905, was presented to the Royal Society in 1908 and published in the book *Canterbury Puzzles*.

Cut the four puzzle pieces with only three cuts.

Arrange these pieces so that you can obtain an equilateral triangle... and conversely, put the pieces back together to obtain a square…

Have fun!

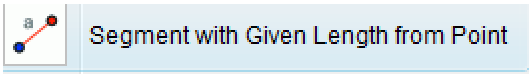
PART 2: DO THE DUDENEY

Assignment 2A: Find a construction to cut the equilateral triangle into four pieces and form a square with the same area. Use the Internet for your research.

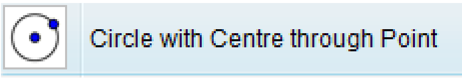
Assignment 2B: Make this simplified construction with the GeoGebra program.
You can use this program online via www.geogebra.org

Start GeoGebra and open a new worksheet.

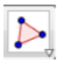
Draw a segment with starting point A and length 2.

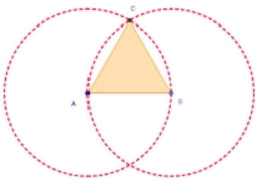


Construct an equilateral triangle by drawing a circle with centre A and passing through B, and a second circle with centre B and passing through A.

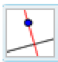


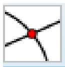
Determine the intersections of two circles with the command 

Draw a polygon (triangle) with points A, B and C 



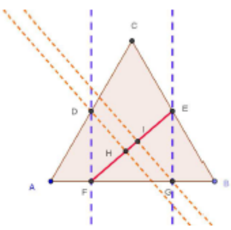
Determine the midpoint D of the side AC and the midpoint E of the side BC.
The length of the segment DE is equal to one unit.

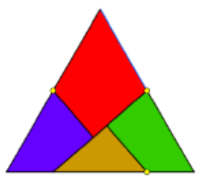
Draw the perpendicular lines from D and E on the side AB 

Determine the intersection points F and G on the side AB 

Draw the segment EF. This is the approximate length of the side of the square required.

Draw the perpendiculars from D and E to this segment and determine the intersection points H and I.





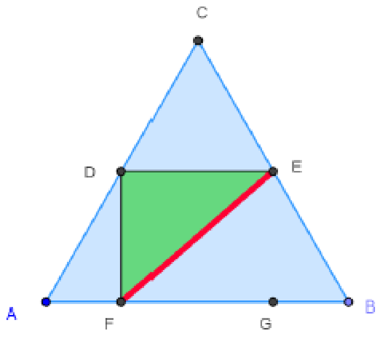
PART 3: CALCULATIONS

Assignment 3A: Calculate the area of the triangle and the length of the constructed side of the square

Assignment 3B: Compare with the actual area of the square.

Solution 3A

It is assumed that the length of side EF is equal to the length of the side of the required square (the same area as the area of the given triangle).
We calculate the length of EF and assume here that the length of the side of the given triangle is equal to 2.



One can prove that the height of this triangle is equal to $\sqrt{3}$
and the length of the segment is …

$$DF = \frac{\sqrt{3}}{2}$$

Apply the Pythagorean theorem to the triangle DEF.

$$(FE)^2 = (DF)^2 + (DE)^2$$

$$(FE)^2=(DF)^2+(DE)^2$$

$$(FE)^2=\left(\frac{\sqrt{3}}{2}\right)^2+1^2$$

$$(FE)^2=\frac{3}{4}+1$$

$$FE=\sqrt{\frac{7}{4}}=\frac{\sqrt{7}}{2}$$

Solution 3B

In this case, the area of the square is 7/4

This is in contradiction with the correct calculation of the area of the square, which is

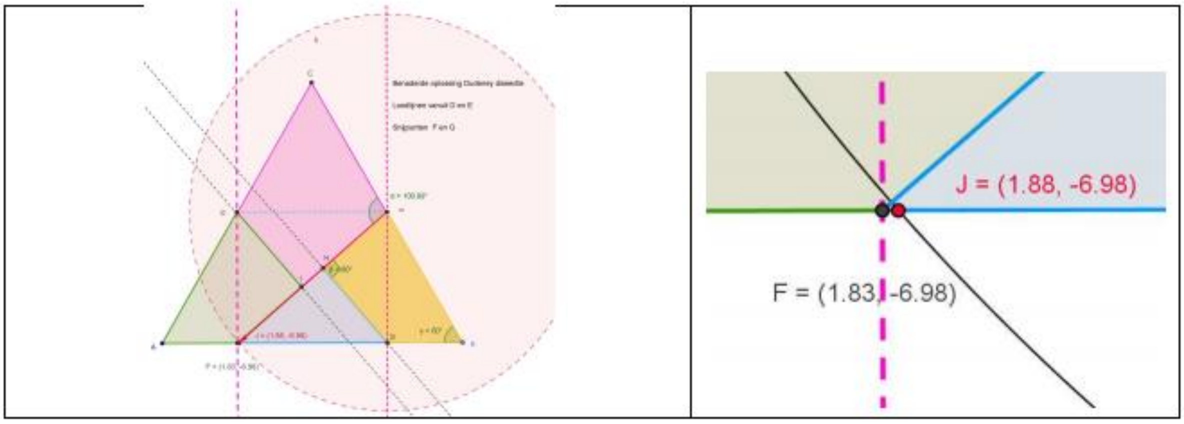
$7/4 \neq \sqrt{3}$

CONCLUSION AND REVISION

The exact value of the surface √3 is different from the estimation 7/4

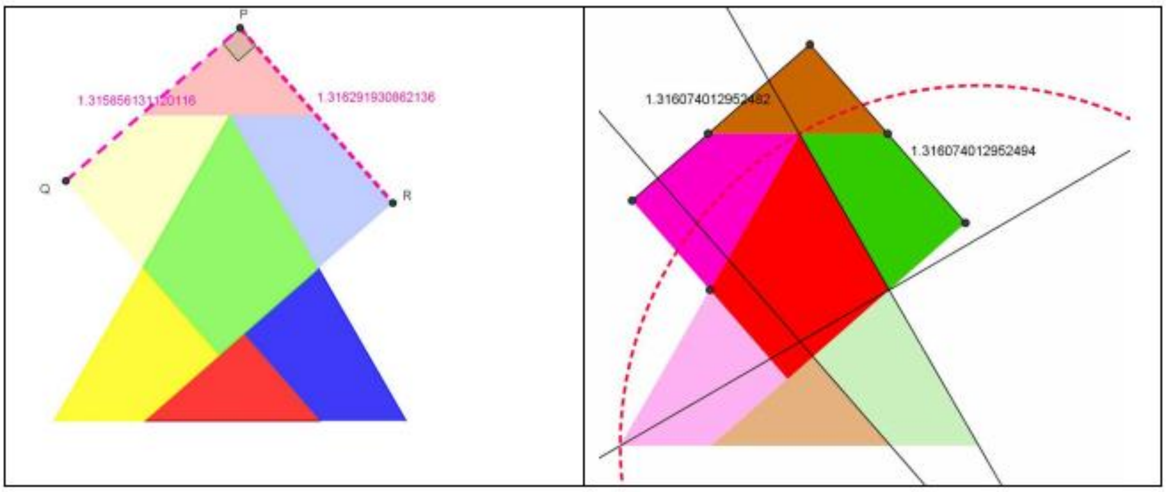
Through calculation it becomes clear that the proposed construction is a good approximation of the problem, but that it was not 100% correct. It is possible to demonstrate this minimal difference with the geometry program GeoGebra.

First method:
By using on the one hand the construction that is an approximate solution of the problem (perpendiculars from the midpoints of two sides) and on the other hand the exact length of the side to fit (from point E a circle of radius √3) one will see that the intersection F does not coincide with J. The abscissas are different (though the difference is minimal...).



Second Method:

After transforming the triangle to a square, the lengths of the sides are not equal!



PART 4: FOLLOW UP

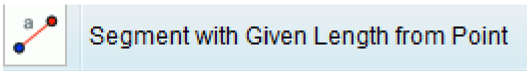
THE CORRECT SOLUTION AND CONSTRUCTION

Consider an equilateral triangle with side 2 (for simplicity of calculation).
The aim is to cut this triangle in 4 pieces to form a square of the same area.
The problem can be expressed as the construction of the side Z of a square the area of which equals that of the given triangle.

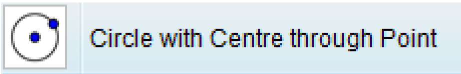
Assignment 4A: Construction of an equilateral triangle with side 2.

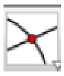
Start GeoGebra and open a new worksheet.

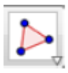
Draw a segment with start point A and length 2.

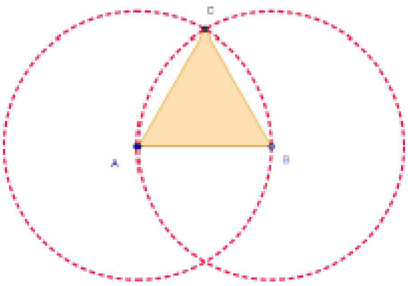


Construct an equilateral triangle by drawing a circle with centre A and passing through B and a second circle with centre B and passing through A.

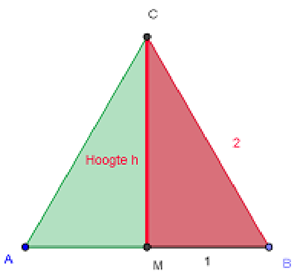


Determine the intersections of the two circles with 

Draw a polygon (triangle) with points A, B and C 



Assignment 4B: Calculation of area of the triangle with side 2.

	Pythagorean theorem $(MB)^2 + (CM)^2 = (BC)^2$ $h^2 = 2^2 - 1^2$ $h^2 = 3$ $h = \sqrt{3}$ S = Area triangle = half of base times height $S = \frac{2\sqrt{3}}{2} = \sqrt{3}$

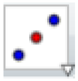
Assignment 4C:

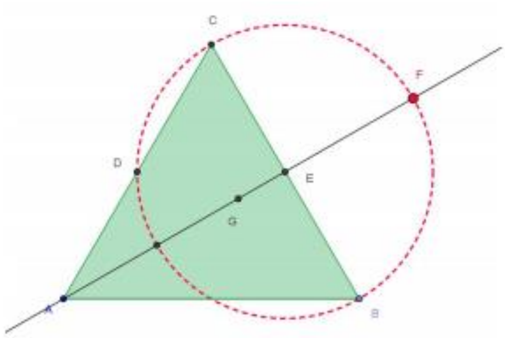
Calculating the length Z of the side of the square with the same area S.

It should be $Z^2 = \sqrt{3}$, which shows that $Z = \sqrt[4]{3}$

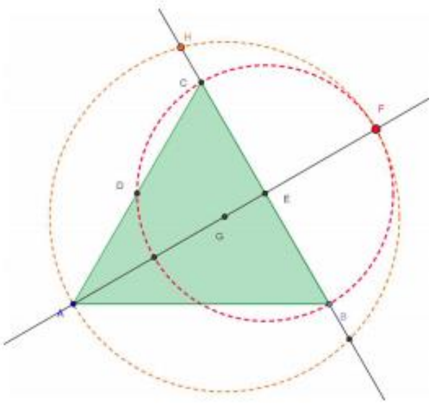
Assignment 4D

Precise construction of a square using GeoGebra with side Z and the 4 puzzle pieces.

- Determine the midpoints D and E of the sides AC and BC with .
- Draw the straight line connecting A and E.
- Draw a circle with centre E and passing through C.
- Determine the midpoint G of the segment AF

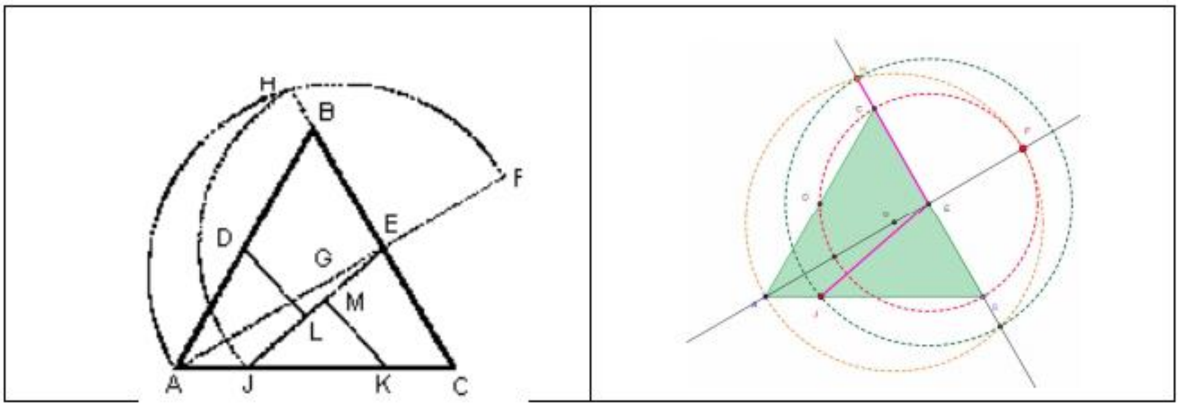


- Draw a circle with centre G and passing through F.
- Determine the intersection H of the straight line connecting A and C in the previous circle.

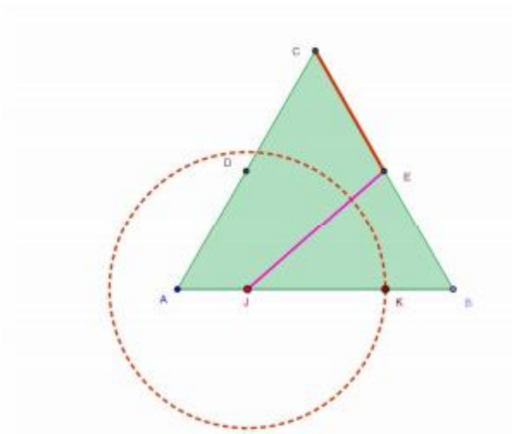


- Draw a circle with centre E and radius of the length of the segment EH (this shows the length of the required square).
- Determine the intersection J of this circle with the side AB.
- Draw the segment EJ = side of square.

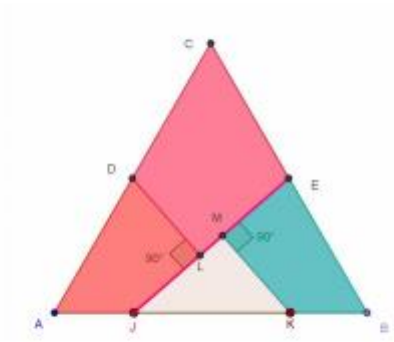
In the book *Canterbury Puzzles*, there is a puzzle picture with this construction (without explanation)



- Remove the construction lines.
- Draw a segment EC and a circle with centre Y and radius EC (use the compass button).



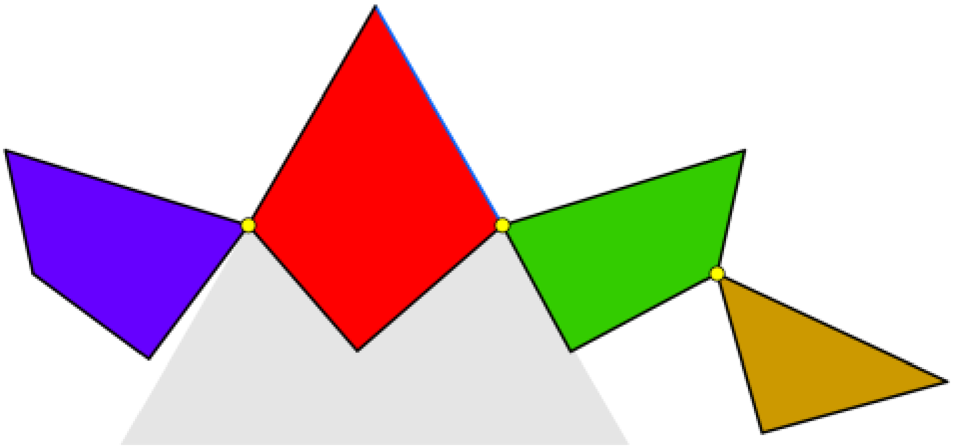
- Draw the perpendicular line on EJ from D and K. Name the intersection points L and M.
- The 4 polygons are AJLD, JKM, KBEM and LECD.



PART 6: ANIMATION OF HINGE PUZZLE

In this final section, we demonstrate an animation created using GeoGebra to illustrate the hinge puzzle.
Of course, there are several ways to form a square with a hinged triangle. The method used in this case is a slider (0° to 180°) and rotations.

With the first option, it is quite simple to indicate the vertices of the rotations.



If one also wants to rotate the third triangle anticlockwise, this requires a bit more creative reflection.